

PROBABILITY**❖ Short Answer Type Question**

- Two dice are thrown once. If it is known that the sum of the numbers on the dice was less than 6 the probability of getting a sum 3 is
(a) $1/18$ (b) $5/18$
(c) $1/5$ (d) $2/5$
- Events E and F are given to be independent. Find $P(F)$ if it is given that $P(E) = 0.60$ and $P(E \cap F) = 0.35$.
- Compute $P(A/B)$, if $P(B) = 0.5$ and $P(A \cap B) = 0.32$.
- Evaluate $P(A \cup B)$, if $2P(A) = P(B) = 5/13$ and $P(A/B) = 2/5$
- If $P(A) = 0.4$, $P(B) = p$ and $P(A \cup B) = 0.6$. Find the value of p , if A and B are independent events.

❖ Long Answer I / Long Answer II Type

- In a class, 40% students study Statistics, 25% Mathematics and 15% both Mathematics and Statistics. One student is selected at random. Find the probability that
 - he studies Statistics, if it is known that he studies Mathematics.
 - he studies Mathematics, if it is known that he studies Statistics.
- In a class, 35% of the students are poor, 20% are meritorious and 15% are both poor and meritorious. One student is selected at random. Find the probability that
 - he is poor, if it is known that he is meritorious
 - he is meritorious, if it is known that he is poor.
- A pair of dice is thrown. Find the probability of getting 7 as the sum, if it is known that second dice always exhibits a prime number.
- Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail then throw a die. Find the conditional probability of the event 'the die shows a number greater than 4' given that "there is at least one tail".

14. Mother, father and son line up at random for a family picture. If E is the event "son on one end" and F is the event "Father in middle", find $P(F|E)$.
15. Probabilities of solving a specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly one of them solves the problem.
16. A and B throw a pair of dice alternately. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. A begins the game, show that odds in favour of A are 30:31.
17. A bag contains tickets numbered 1, 2, 3, 50 out of which five tickets x_1, x_2, x_3, x_4, x_5 are drawn at random and arranged in ascending order of magnitude $x_1 < x_2 < x_3 < x_4 < x_5$. What is the probability that $x_3 = 30$?
18. The probability of a student A passing an examination is $\frac{3}{5}$ and of student B is $\frac{4}{5}$. Assuming that the two events "A passes", "B passes" as independent. Find the probability of (i) both the students passing the examination (ii) only A passing the examination (iii) only one of them passing the examination (iv) none of them passing the examination.
19. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown, If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black.
20. Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?

21. Out of 9 outstanding students of a school, there are 4 boys and 5 girls. A team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.
22. A speaks truth in 60% of the cases, while B in 90% of the cases. In what per cent of cases are they likely to contradict each other in stating the same fact? In the cases of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of cases than A?
23. A class consists of 80 students. 25 of them are girls and remaining boys, 10 of them are rich and remaining poor, 20 of them are fair complexioned and others not. What is the probability of selecting a fair complexioned rich girl?
24. A bag contains 3 red and 7 black balls. 2 balls are selected at random without replacement. If the second selected is given to be red, what is the probability that the first selected is also red?
25. A and B throw a pair of dice alternately. A wins the game if he gets a total of 6 and B wins if she gets a total of 7. If A starts the game, find the probability of winning the game by A in third throw of pair of dice.

PRACTICE QUESTIONS

❖ Long Answer I / Long Answer II Type

1. Two groups are competing for the positions on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if second group wins. Find the probability that the new product introduced was by the second group.
- [NCERT]
2. There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up tails 25% of the times

and the third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows head, what is the probability that it was from the two headed coin?

3. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?
[NCERT: Foreign 2011]
4. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?
[NCERT: AI 2012]
5. Suppose that the reliability of a HIV test is specified as follows: Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% the test are judged HIV-ve but 1% are diagnosed as showing HIV +ve. From a large population of which only 0.1% have HIV, one person is selected at random given the HIV test, and the pathologist reports him/he as HIV +ve. What is the probability that the person actually has HIV?
[NCERT]
6. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack 3 cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade.
[Delhi 2014]
7. In a bulb factory, machines A, B and C manufacture 60%, 30% and 10% bulbs respectively. 1%, 2% and 3% of the bulbs produced respectively by A, B and C are found to be defective. A bulb is picked up at random from the product and is found to be defective. Find the probability that this bulb was produced by the machine A.

8. Bag I contains 3 red and 4 black balls and bag II contains 4 red and 5 black balls. One ball is transferred from bag I to bag II and then a ball is drawn from bag II at random. The balls so drawn is found to be red in colour.

Find the probability that the transferred ball is black

[Foreign 2011]

9. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident for them are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver or a car driver?

[Foreign 2014]

RANDOM VARIABLES AND ITS PROBABILITY DISTRIBUTIONS

1. A discrete random variable is a variable which takes integral values depending upon the outcomes of the experiment.
2. A probability distribution represents that how probability of an experiment is distributed over different exhaustive events of the experiment. If x_1, x_2, \dots, x_n are the possible real number values associated to different exhaustive events of an experiment and P_1, P_2, \dots, P_n are their respective probabilities, then distribution is represented as
3. For a given probability distribution
 - (i) Mean (μ) = $E(X) = \sum_{i=1}^n x_i p_i$; μ is also called expected value of X, $E(X)$.
 - (ii) Variance (σ^2) = $\sum_{i=1}^n x_i^2 p_i - \mu^2$
 - (iii) Standard deviation $\sigma = \sqrt{\text{Variance}}$
4. A company has two plants to manufacture TVs. The first plant manufactures 70% of the TVs and the rest are manufactured by the other plant. 80% of the TVs manufactured by the first plant are rated of standard quality, while that of the second plant only 70% are of standard quality. If a TV chosen at random is found to be of standard quality, find the probability that it was produced by the first plant.

5. A committee of 4 students is selected at random from a group consisting of 8 boys and 4 girls. Given that there is at least one girl in the committee, calculate the probability that there are exactly 2 girls in the committee. **[NCERT Exemplar]**
6. An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. Show that the probability of drawing a white ball does not depend on k .
7. A bag contains $(2n + 1)$ coins, out of which n coins have head on both the sides and rest are fair coins. A coin is selected at random and is tossed, if it results in a head with probability $\frac{31}{42}$ find n .
8. There are three coins. One is a two headed coin (having heads on both faces), another is a biased coin that comes up heads 75% of the times and the third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads. What is the probability that it was the two-headed coin? **[NCERT; Foreign 2011]**
9. A bag I contains 5 red and 4 white balls and a bag II contains 3 red and 3 white balls. Two balls are transferred from the bag I to the bag II and then one ball is drawn from the bag II. If the ball drawn from the bag II is red, then find the probability that one red ball and one white ball are transferred from the bag I to the bag II. **[CBSE Sample Paper 2016]**
10. Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females. **[NCERT; Delhi 2017]**
11. Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

12. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain 4 grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance?
13. In a shop X, 30 tins of pure ghee and 40 tins of adulterated ghee which look alike, are kept for sale while in shop Y, similar 50 tins of pure ghee and 60 tins of adulterated ghee are there. One tin of ghee is purchased from one of the randomly selected shops and is found to be adulterated. Find the probability that it is purchased from shop Y.
14. Two integers are selected at random from integers 1 to 11. If the sum of integers chosen is even, find the probability that both numbers are odd.
15. Bag I contains 3 red and 4 black balls and bag II contains 4 red and 5 black balls. Two balls are transferred at random from bag I to bag II and then a ball is drawn from bag II. The ball so drawn is found to be red in color. Find the probability that the transferred balls were both black. <https://loyaleducation.org> [Delhi 2012]
16. By examining the chest X-ray, the probability that TB is detected when a person is actually suffering from it is 0.99. The probability that the diagnosis incorrectly that a person has TB on the basis of X-ray is 0.001. In a certain city, 1 in 1000 persons suffers from TB. A person is selected at random and is diagnosed to have TB. What is the chance that he actually has TB?
17. A factory has three machines X, Y and Z producing 1000, 2000 and 3000 bolts per day respectively. The machine X produces 1%, Y produces 1.5% and Z produces 2% defective bolts. At the end of a day, a bolt is drawn at random and is found defective. What is the probability that this defective bolt has been produced by the machine X?
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